General Heart Construction

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Aim

Give an abstract construction

Triangulated category $\mathcal{C}$ ⇝ Abelian category '*heart*'

Setting

$(\mathcal{U}, \mathcal{V})$: a pair of subcategories of $\mathcal{C}$

- $\mathrm{Ext}^1(\mathcal{U}, \mathcal{V}) = 0$
- $\forall C \in \mathrm{Ob}(\mathcal{C})$, $\exists$dist.$\triangle$

$$U \to C \to V[1] \to U[1]$$

$(U \in \mathrm{Ob}(\mathcal{U}), V \in \mathrm{Ob}(\mathcal{V}))$

generalizing both cases:

1. t-structure $(\Leftrightarrow \mathcal{V} \subseteq \mathcal{V}[1])$

$(\mathcal{T}^{\leq 0}, \mathcal{T}^{\geq 0})$ ⇝ $\mathcal{T}^{\leq 0} \cap \mathcal{T}^{\geq 0}$

2. cluster tilting subcategory $(\Leftrightarrow \mathcal{V} = \mathcal{U})$

$\mathcal{T} \subseteq \mathcal{C}$ ⇝ $\mathcal{C}/\mathcal{T}$

(※Any subcategory is assumed to be full, additive, thick and replete.)

Theorem 1 (N-)

$\mathcal{C} \supseteq \mathcal{H} :=$

Full subcategory

$C \in \mathrm{Ob}(\mathcal{C})$ admitting dist.$\triangle$'s

$$V \to W_1 \to C \to V[1]$$

$$U[-1] \to C \to W_2 \to U$$

$(\exists U \in \mathcal{U}, \exists V \in \mathcal{V}, \exists W_1, W_2 \in \mathcal{U} \cap \mathcal{V})$

Then

$$\underline{\mathcal{H}} := \mathcal{H}/(\mathcal{U} \cap \mathcal{V}) \quad \text{is abelian.}$$

generalize 1 & 2

Theorem 2 (Abe, N-)

$$\exists H : \mathcal{C} \longrightarrow \underline{\mathcal{H}} \quad \text{a cohomological functor}$$

Composition of some adjoints and the quotient